Multirate Signal Processing

Multirate : Changing the **Sampling Rate** of the Discrete Time Signal.

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Need for Multirate

Digital Audio : Three different sampling rates are employed

- Broadcasting : 32 KHz
- Digital Compact Disc : 44.1 KHz
- Digital Audio Tape : 48 KHz

Digital Video :

- Luminance signal is sampled at 13.5 MHz
- Color difference signal is sampled at 6.75 MHz

Sampling rate of

- NTSC Composite signal : 14.31818 MHz
- PAL Composite signal : 17.73447 MHz

- *Over sampling A / D Converter* :

Sample the signal at much higher rate than Nyquist rate and then decimate (lower the sampling rate) it.

Eliminates the need of very sharp cut- off frequency antialiasing filter

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Digital Communication :

Trans multiplexers

Communication Receivers

Biomedical :

Narrow band filter for Fetal ECG & EEG

Speech Processing :

To reduce storage space and transmission rate

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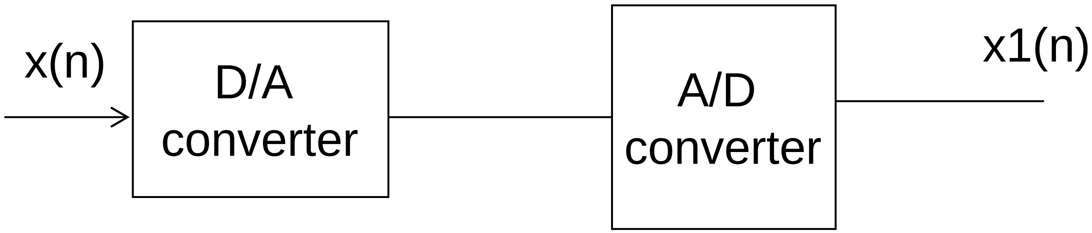
The process of converting a signal from a given rate to a different rate is called sampling rate conversion.

Systems that employ multiple sampling rates in the processing of digital signals are called multirate digital signal processing systems.

Can be accomplished in two ways:



Adv: New sampling rate can be arbitrarily selected

Disadv: Signal distortion is introduced by
-D/A converter in signal reconstruction
-Quantization effects in A/D converter

Changing the sampling rate in digital domain --Multirate

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Fundamental Operations in multirate signal processing are

Decimation (Down sampling)
Interpolation (Up Sampling)

DECIMATION

Decimation is the process of decreasing the sampling rate by a factor of M i.e. from F_s to F_s / M

Down sampling : by a factor of M is achieved by discarding **M-1** samples for every M samples.

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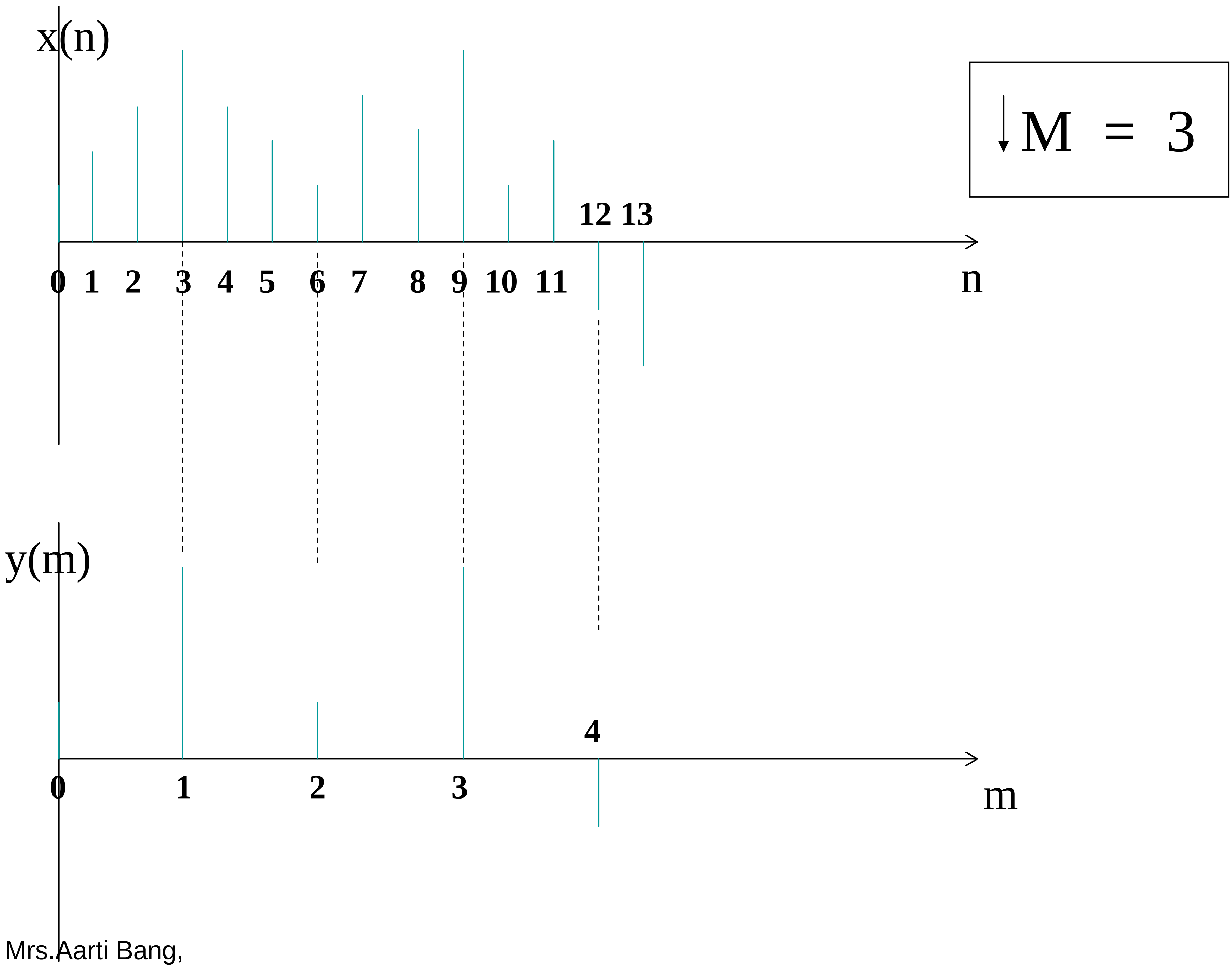

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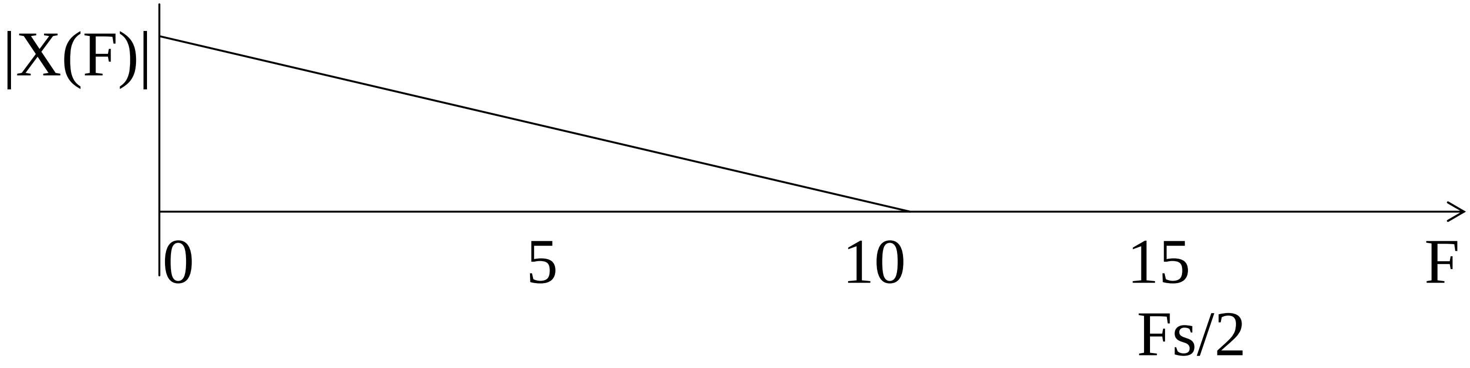
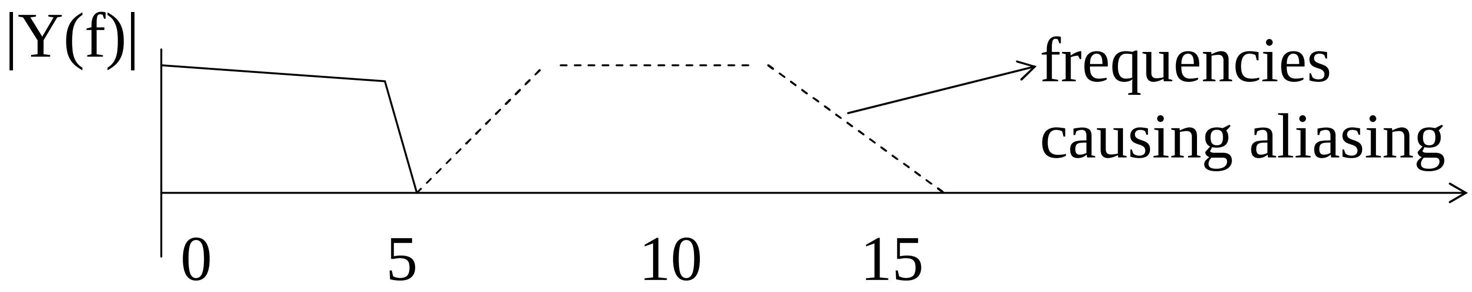
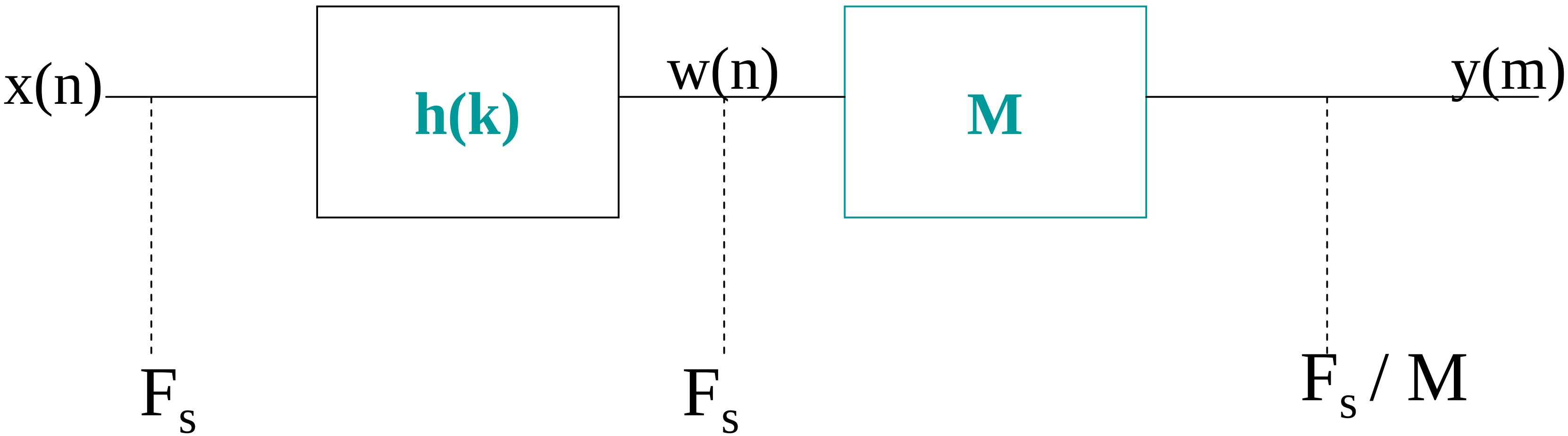


M = 3



This combined operation of filtering and down sampling is called as **DECIMATION**



Block diagram of Decimator

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The rate compressor reduces the sampling rate from F_s to F_s / M.

To prevent aliasing at lower rate, digital filter is used to band limit the i/p signal to less than $F_s / 2M$. (new folding frequency)

Sampling rate reduction is achieved by discarding M-1 samples for every M samples of filtered signal w(n)

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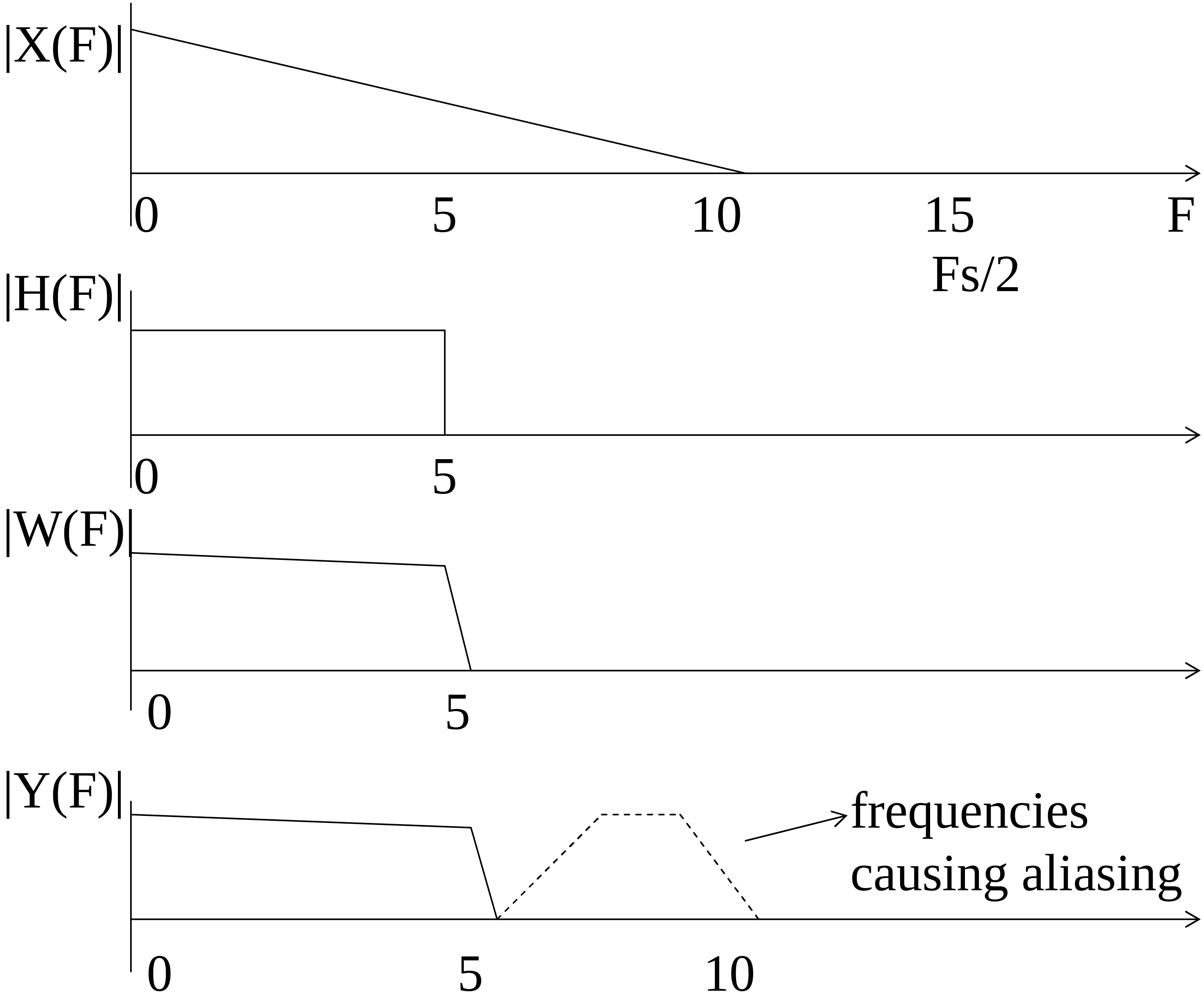

Spectral interpretation of decimation from 30 to 10 Hz

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I/P O/P relationship :

$$y(m) = w(mM) = \sum_{k=-\infty}^{\infty} h(k)x(mM-k)$$

where

$$w(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

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Here $$x(n-n_0) \neq y(n-n_0)$$

Filtering operation is linear & time invariant,
But downsampling is not.
Therefore DECIMATION is a time variant operation

Frequency domain characteristics

Let o/p sampling frequency be F_y or ω_y
I/p sampling frequency be F_x or ω_x
Decimation factor be M

$$\frac{\omega_y}{\omega_x} = M \quad or \quad \omega_y = \omega_x M$$

Thus frequency range $0 <= | \omega_x | <= \pi/M$
is stretched into corresponding
Frequency range of $0 <= | \omega_y | <= \pi/M$

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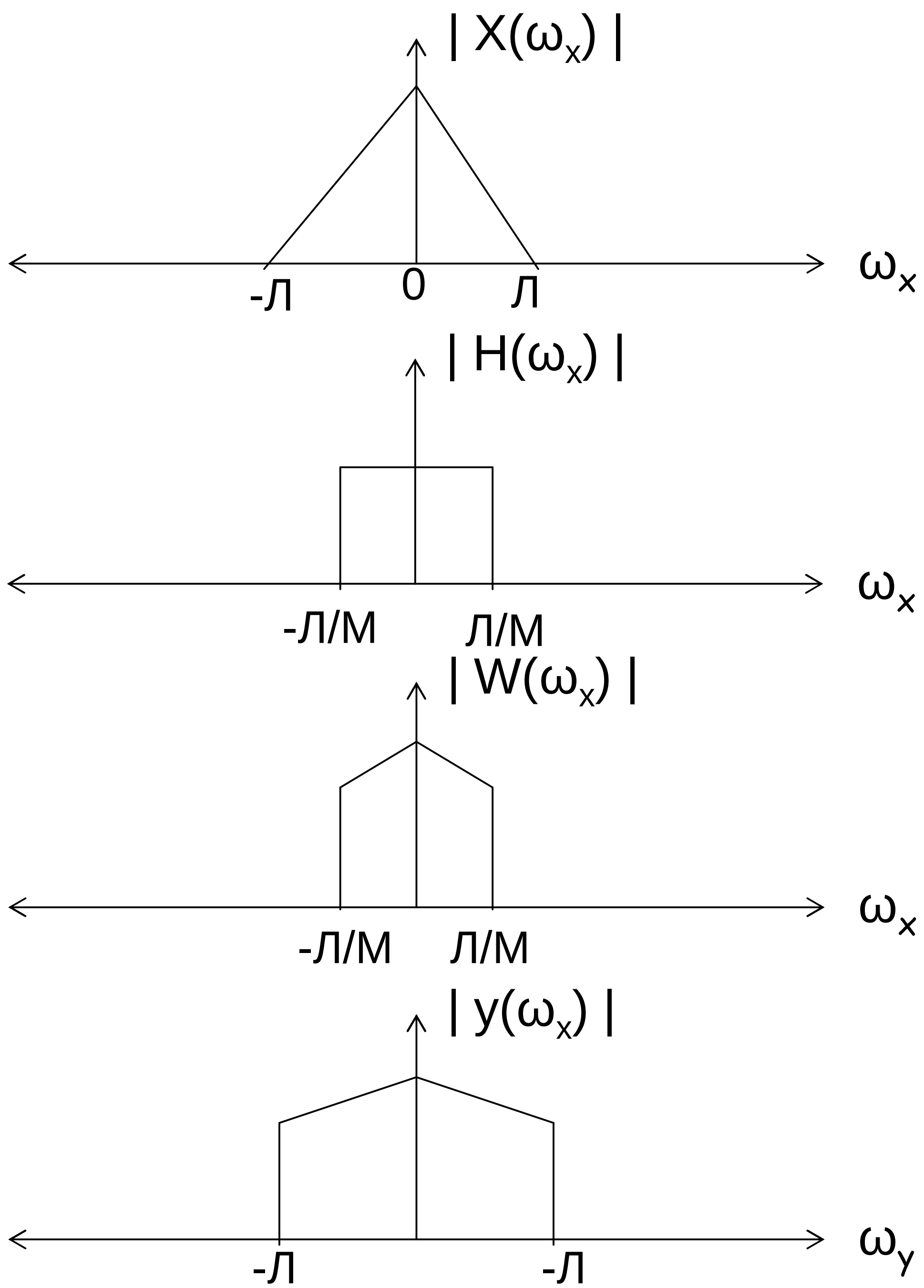

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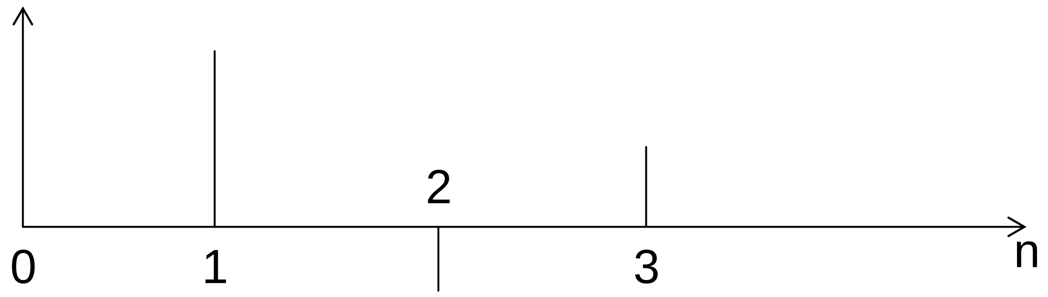
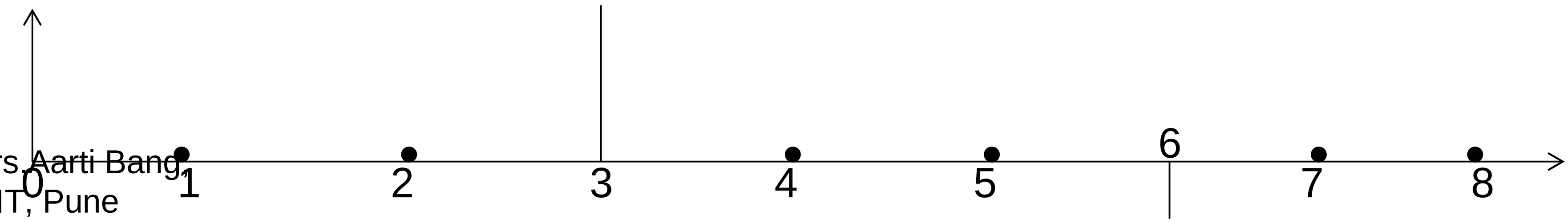
INTERPOLATION

Process of increasing the sampling rate of the signal by a factor of L i.e. from F_s to LF_s

Upsampling by a factor L → inserting L-1 zeros between two samples





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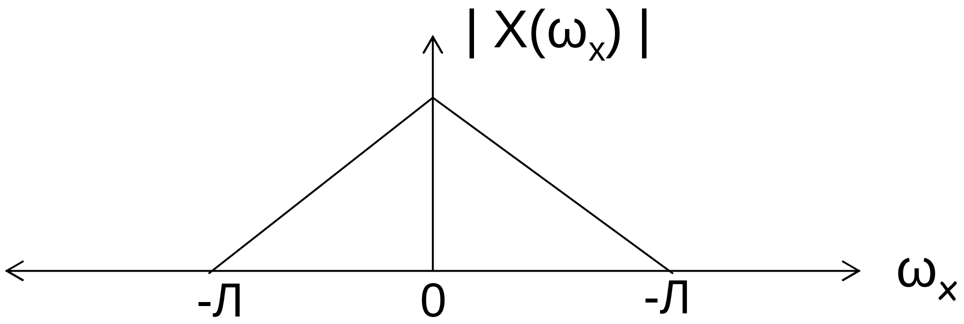
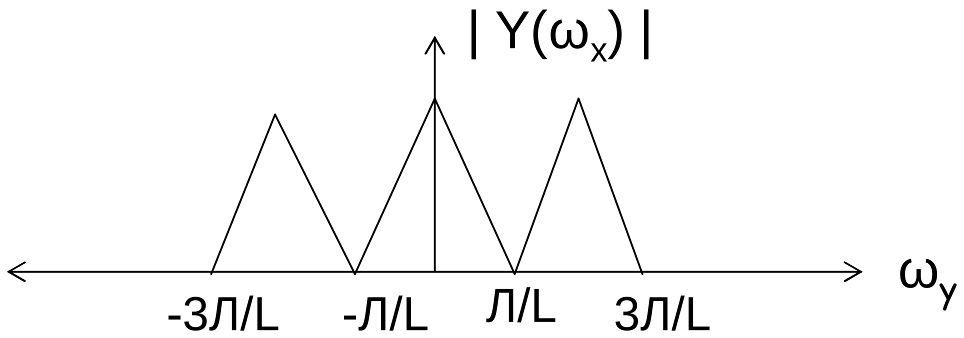
Inserting zeros create image bands.





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Frequency domain characteristics

Let o/p sampling frequency be F_y or ω_y
I/p sampling frequency be F_x or ω_x
Interpolation factor be L

$$\omega_y = \frac{\omega_x}{L}$$

Thus frequency range $0 <= |\omega_x| <= \pi$
is compressed into corresponding
Frequency range of $0 <= |\omega_y| <= \pi/L$
and the o/p spectrum is L fold periodic repitition of i/p spectrum.

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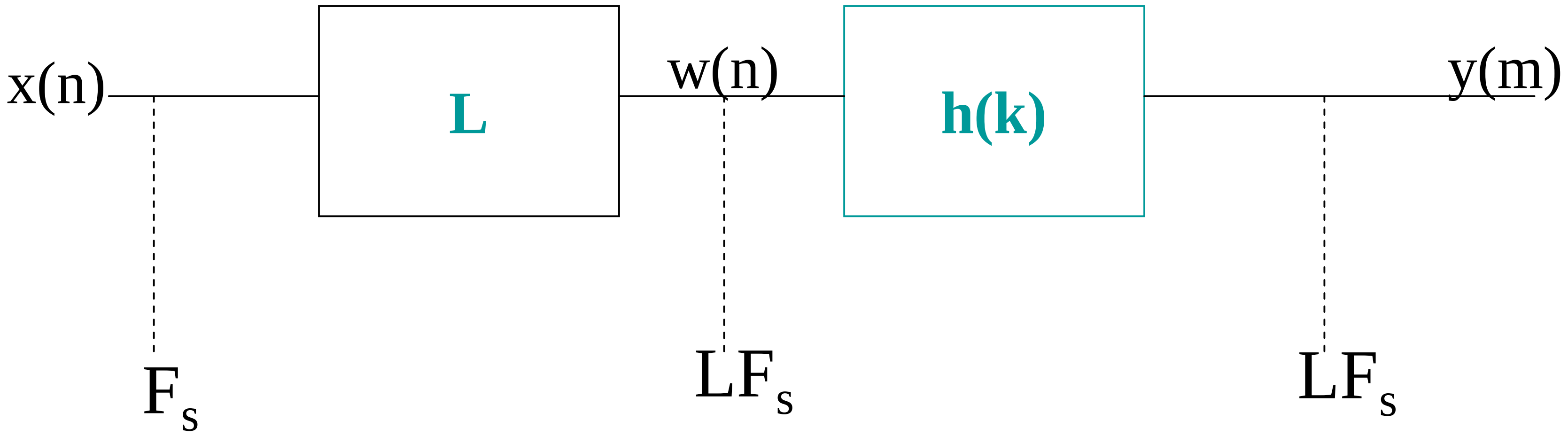
Spectral components of x(n) in the range

$0 <= | \omega y | <= \pi/L$ are unique

Images above $\omega y = \pi/L$ are to be rejected by filtering



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Decimation = Filtering + Downsampling

(antialiasing)

Interpolation = Upsampling + Filtering

(antiimaging)

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Sampling rate conversion

by

Rational / Non Integer factor

Transferring data from Compact Disc to Audio Tape

44.1 KHz → 48 KHz

$$\frac{48}{44.1} = \frac{160}{147} = \frac{L}{M}$$

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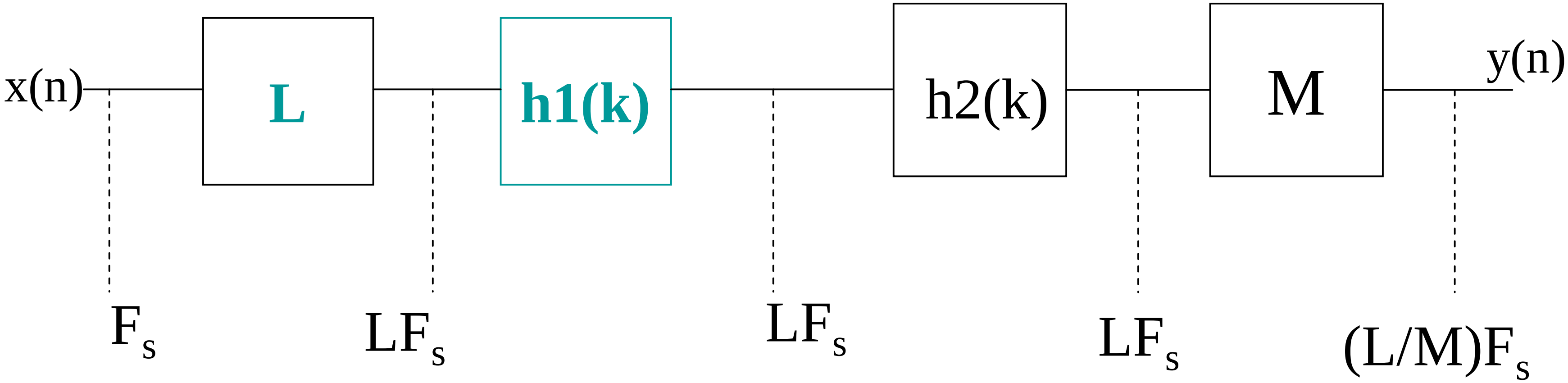

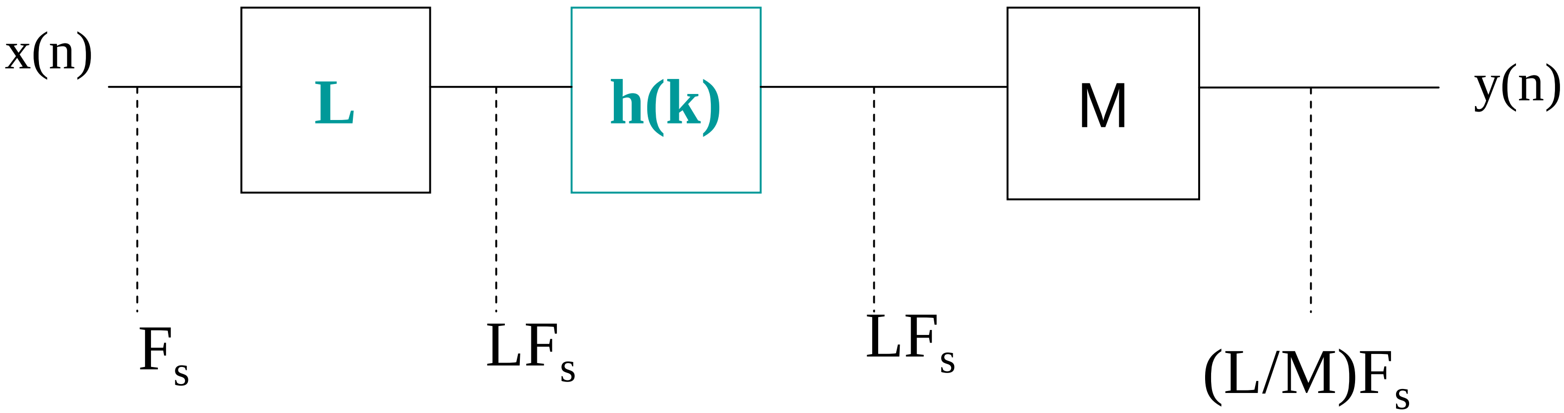

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Specifications of antiimaging filter

$$|H(e^{j\omega})| = L \quad |\omega| \leq \pi / L$$

$$0 \quad \pi / L \leq |\omega| \leq \pi$$

Specifications of antialiasing filter

$$|H(e^{j\omega})| = 1 \quad |\omega| \leq \pi / M$$

$$0 \quad \pi / M \leq |\omega| \leq \pi$$

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Specifications of combined filter

$$\left|H(e^{j\omega})\right| = L \quad |\omega| \leq (\min \pi / M, \pi / L)$$
$$0 \quad otherwise$$

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Multistage approach to sampling rate conversion

High sampling rate conversion →
Efficient to change rate in more no. of stages

$$M = M_1 \, M_2 \, M_3 \ldots \, M_I$$

Reduced computational effort and storage requirements

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Estimation of Filter Order (M) :

Kaiser's formula:

$$M = \frac{-10\log_{10}(\delta_p \delta_s) - 13}{14.6\Delta f}$$

or

$$M = \frac{-10\log_{10}(\delta_p \delta_s) - 13}{14.6(\omega_{st} - \omega_p)/2\pi}$$

M is inversely proportional to transition bandwidth ($\omega_s - \omega_p$) and not on transition band location.

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Estimation of Filter Order (M) :

- Hermann-Rabiner-Chan's Formula:

$$M \cong \frac{D_\infty(\delta_p,\delta_s) - F(\delta_p,\delta_s)[(\omega_s-\omega_p)/2\pi]^2}{(\omega_s-\omega_p)/2\pi}$$

where

$$D_\infty(\delta_p,\delta_s) = [a_1(\log_{10}\delta_p)^2 + a_2(\log_{10}\delta_p) + a_3]\log_{10}\delta_s$$
$$+[a_4(\log_{10}\delta_p)^2 + a_5(\log_{10}\delta_p) + a_6]$$

$$F(\delta_p,\delta_s) = b_1 + b_2[\log_{10}\delta_p - \log_{10}\delta_s]$$

with

$$a_1 = 0.005309,\ a_2 = 0.07114,\ a_3 = -0.4761$$
$$a_4 = 0.00266,\ a_5 = 0.5941,\ a_6 = 0.4278$$
$$b_1 = 11.01217,\quad b_2 = 0.51244$$

Filter specifications for each stage

Passband $0 <= F <= F_p$

Stopband $(F_i – F_s/2M) < F < F_{i-1}/2$ i= 1,2,…..,.,I

Passband ripple δ_p
Stopband ripple δ_s

where F_i → o/p sampling frequency of i^{th} stage
F_{i-1} → i/p sampling frequency of i^{th} stage
F_s → original sampling frequency
F_p → highest frequency of interest

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Determining the no. of stages and decimation factors

Least computational effort → Optimum no. of stages

$$MPS = \sum_{i=1}^{I} N_i F_i \qquad TSR = \sum_{i=1}^{I} N_i$$

where Ni is the no. of filter coefficients per stage

MPS is no. of Multiplications Per Second
TSR is Total Storage Requirement for coefficients

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Advantages of Multistage design:

- Multistage design yields significant reductions in computation & storage requirements compared to single stage.
- Reductions are due to wide transition bands of filters at early stages (even though sampling rates are high) leading to small values of N.
- In the last stage though the transition band is small, the sampling rate is also low, hence filter order (N) is also small as compared to single stage decimation.

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