DFT

and

Spectral Analysis of Signals

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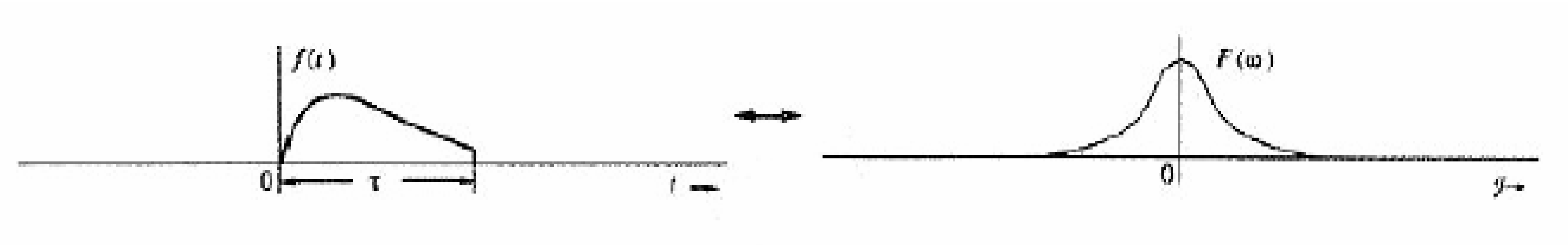
Continuous Time Case : **Fourier Transform**

$$H(F) = \int_{-\infty}^{+\infty} h(t) e^{-j2\pi Ft} dt$$

$$h(t) = \int_{-\infty}^{+\infty} H(F) e^{j2\pi Ft} dF$$



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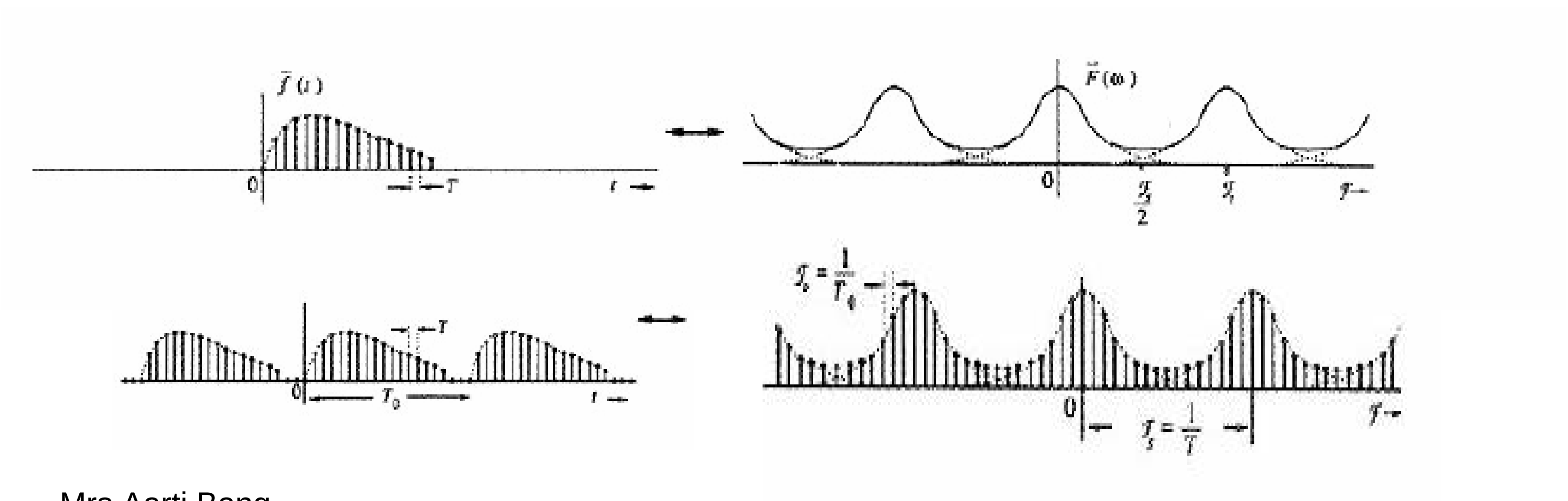
Discrete Time Case : Discrete Time Fourier Transform

$$H(\omega) = \sum_{n=-\infty}^{+\infty} h(n) e^{-j\omega n}$$

$$h(n) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} H(\omega) e^{j\omega n} d\omega$$



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Discrete Fourier Transform

DFT

$$H(k) = \sum_{n=0}^{N-1} h(n) e^{-j2\pi nk/N} \qquad k = 0,1,2\ldots.N-1$$

IDFT

$$h(n) = \frac{1}{N} \sum_{k=0}^{N-1} H(k) e^{j2\pi nk/N} \qquad n = 0,1,2\ldots.N-1$$

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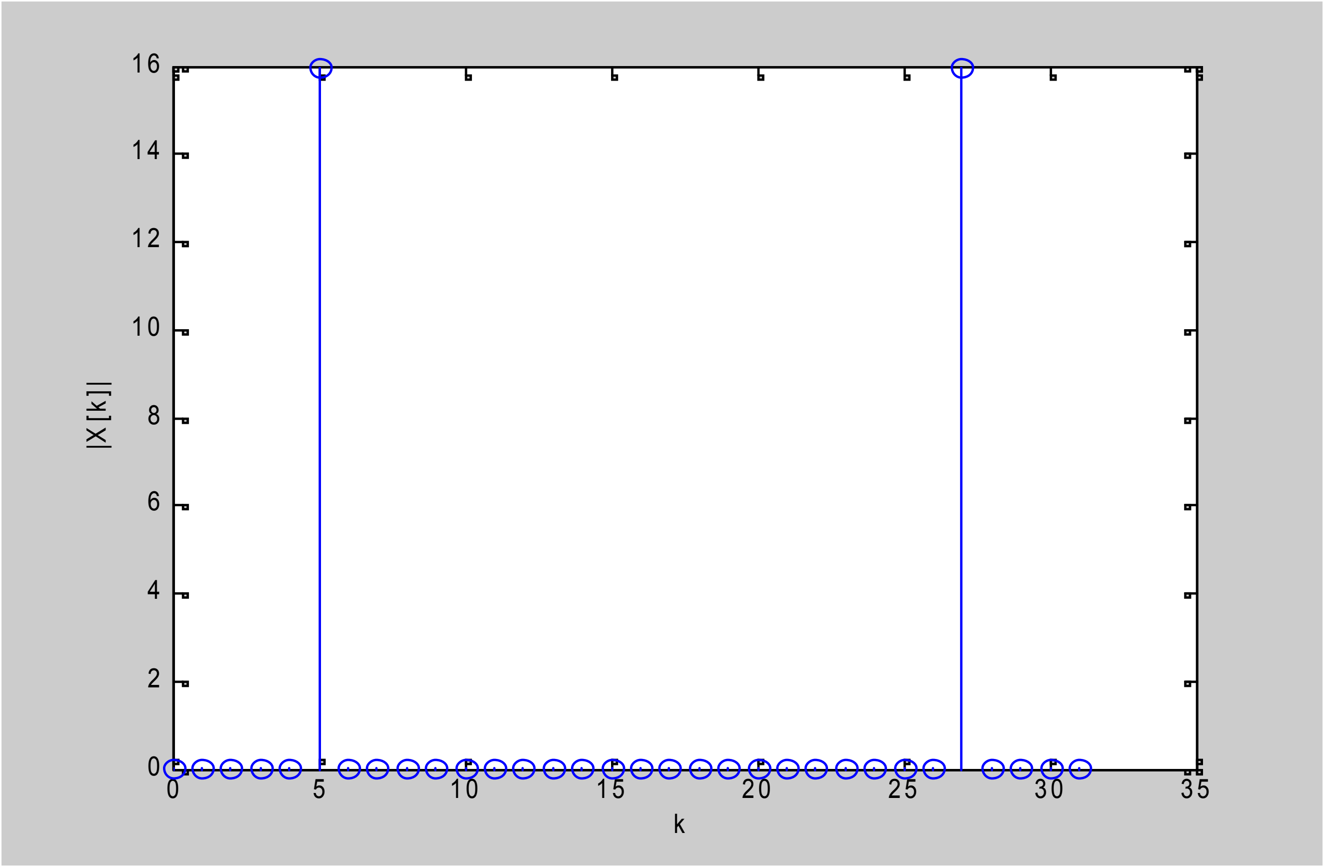
e.g.

F=10 Hz   Fs = 64 Hz   N = 32



x(n) = sin(2*pi*n*10/64)

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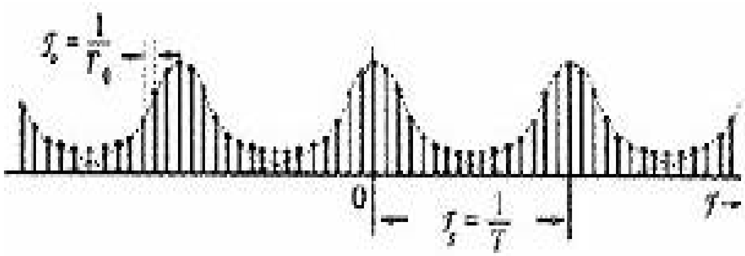
$$H(\omega) = \sum_{n=-\infty}^{+\infty} h(n) e^{-j\omega n}$$

$$\omega = \frac{2\pi k}{N}$$

$$H(k) = \sum_{n=0}^{N-1} h(n) e^{-j2\pi nk/N}$$

$$\frac{2\pi F_k}{Fs} = \frac{2\pi k}{N}$$



$$F_k = \frac{kF_s}{N}$$

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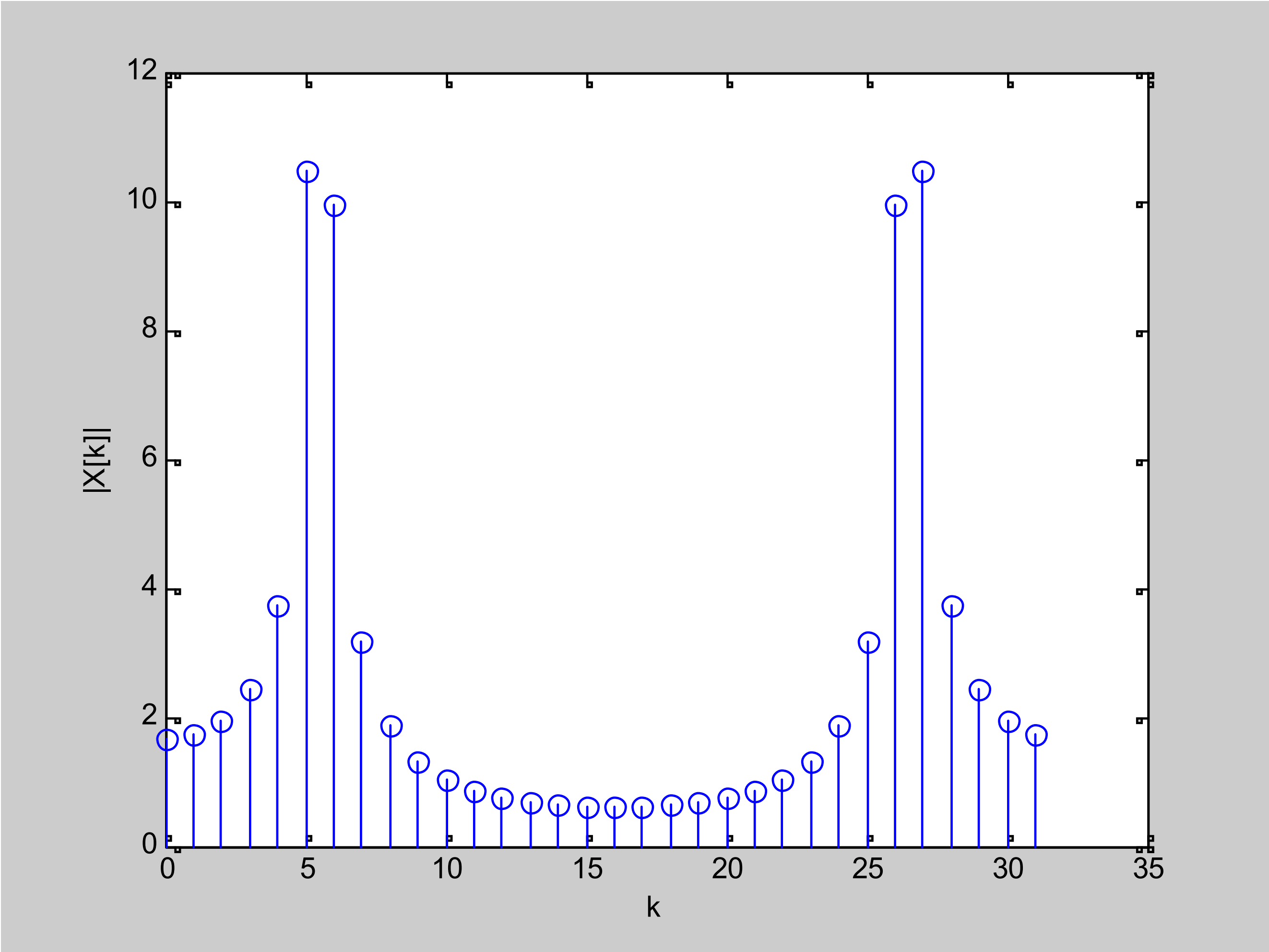
F=11 Hz   Fs = 64 Hz   N = 32



This phenomenon is called **leakage**

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The phenomenon of the spread of energy from a single frequency to many frequency locations is called leakage.

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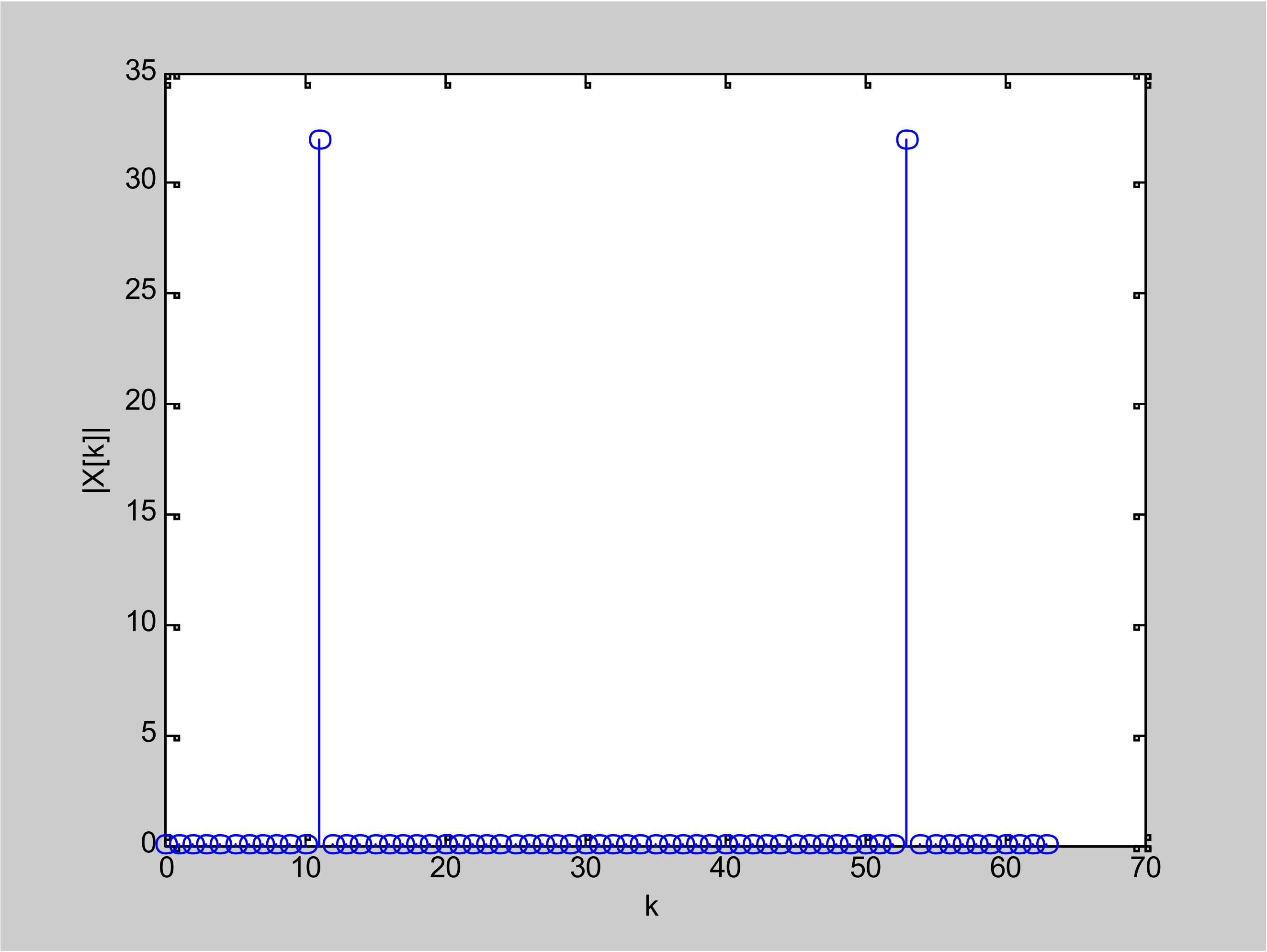
F=11 Hz Fs = 64 Hz N = 64



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Windowing

x^(n) = x(n) x w(n)

where $$w(n) = 0.54 + 0.46\cos\left(\frac{2\pi n}{2N+1}\right)$$

n = 0,1,…, N-1

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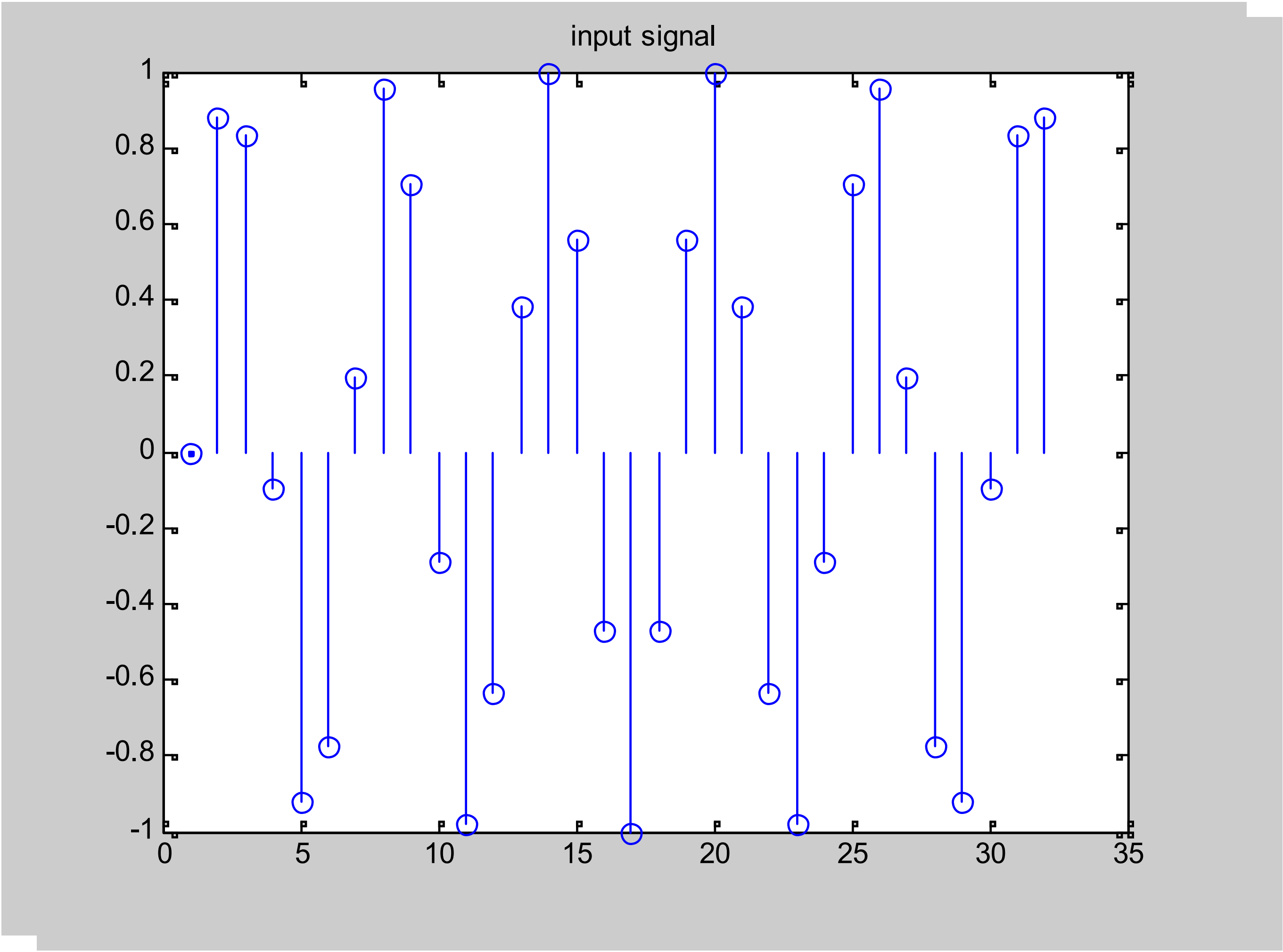

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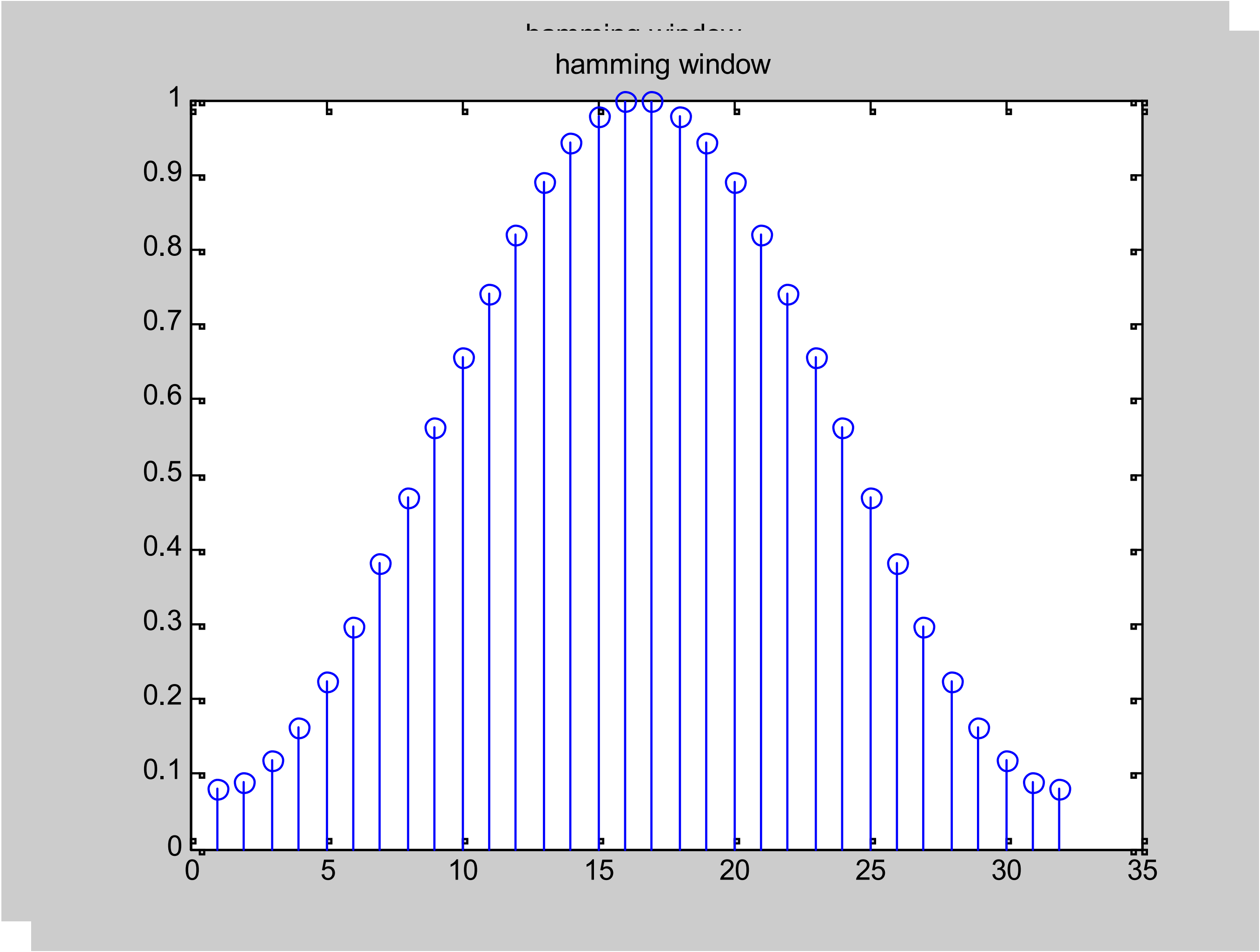

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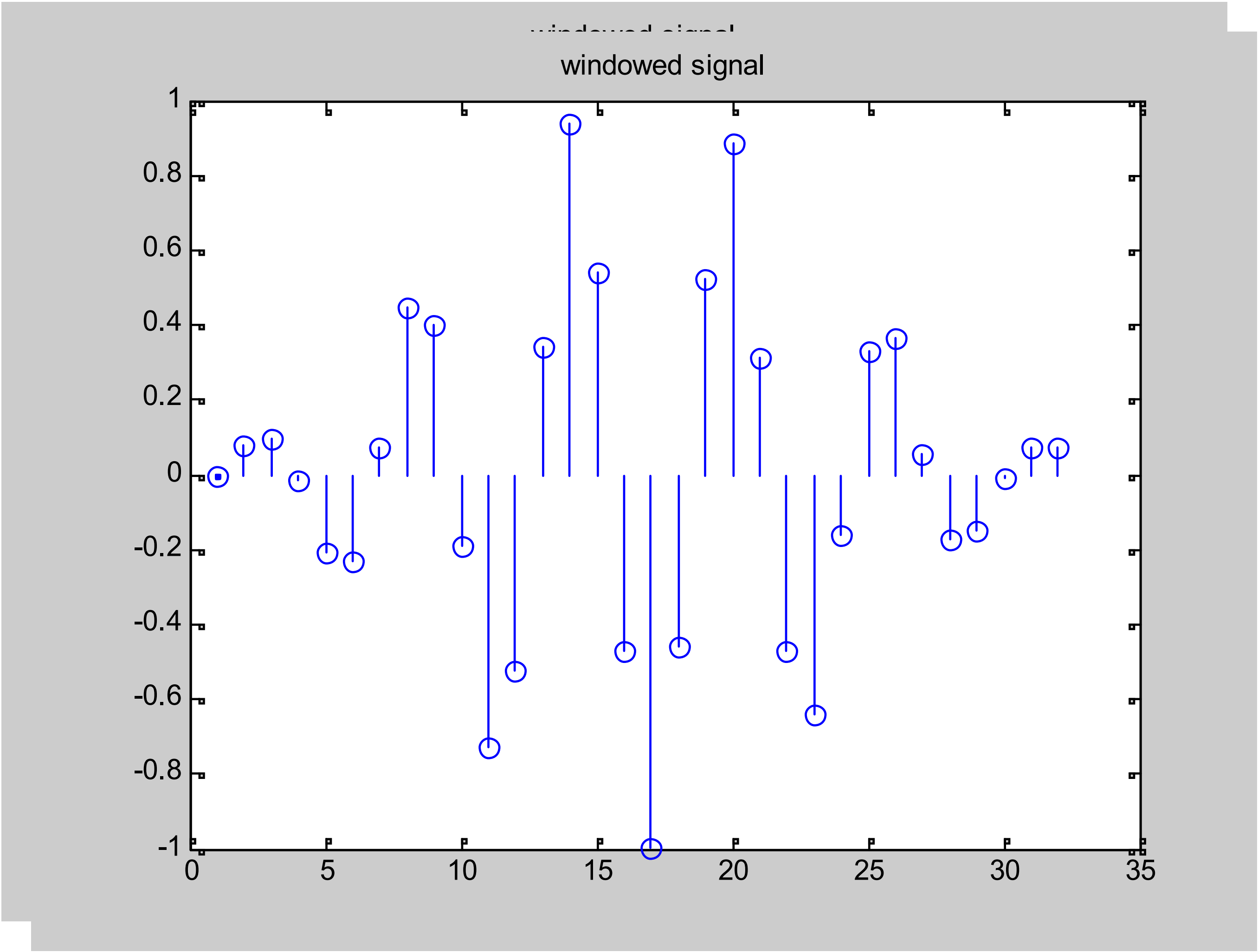

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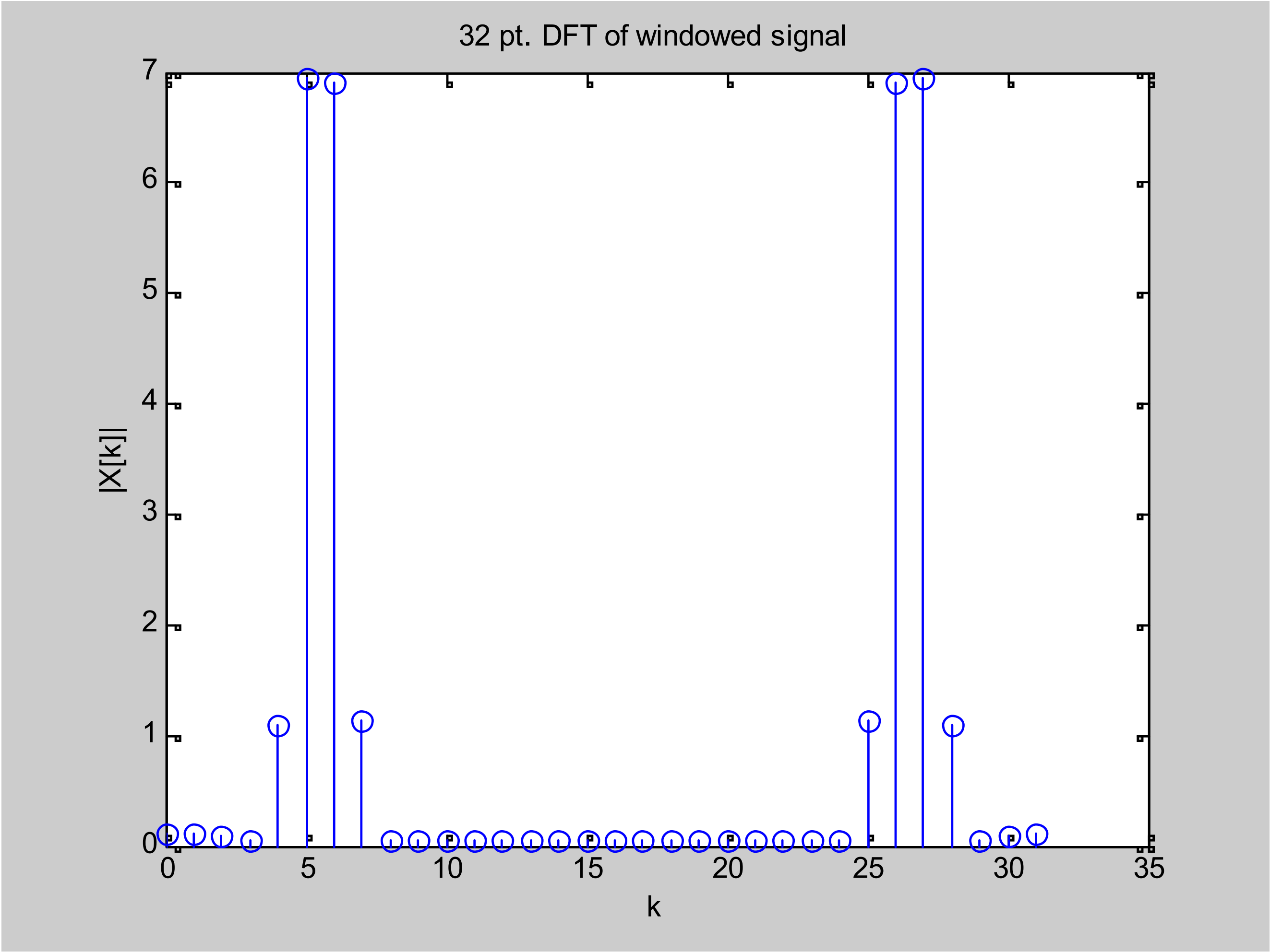

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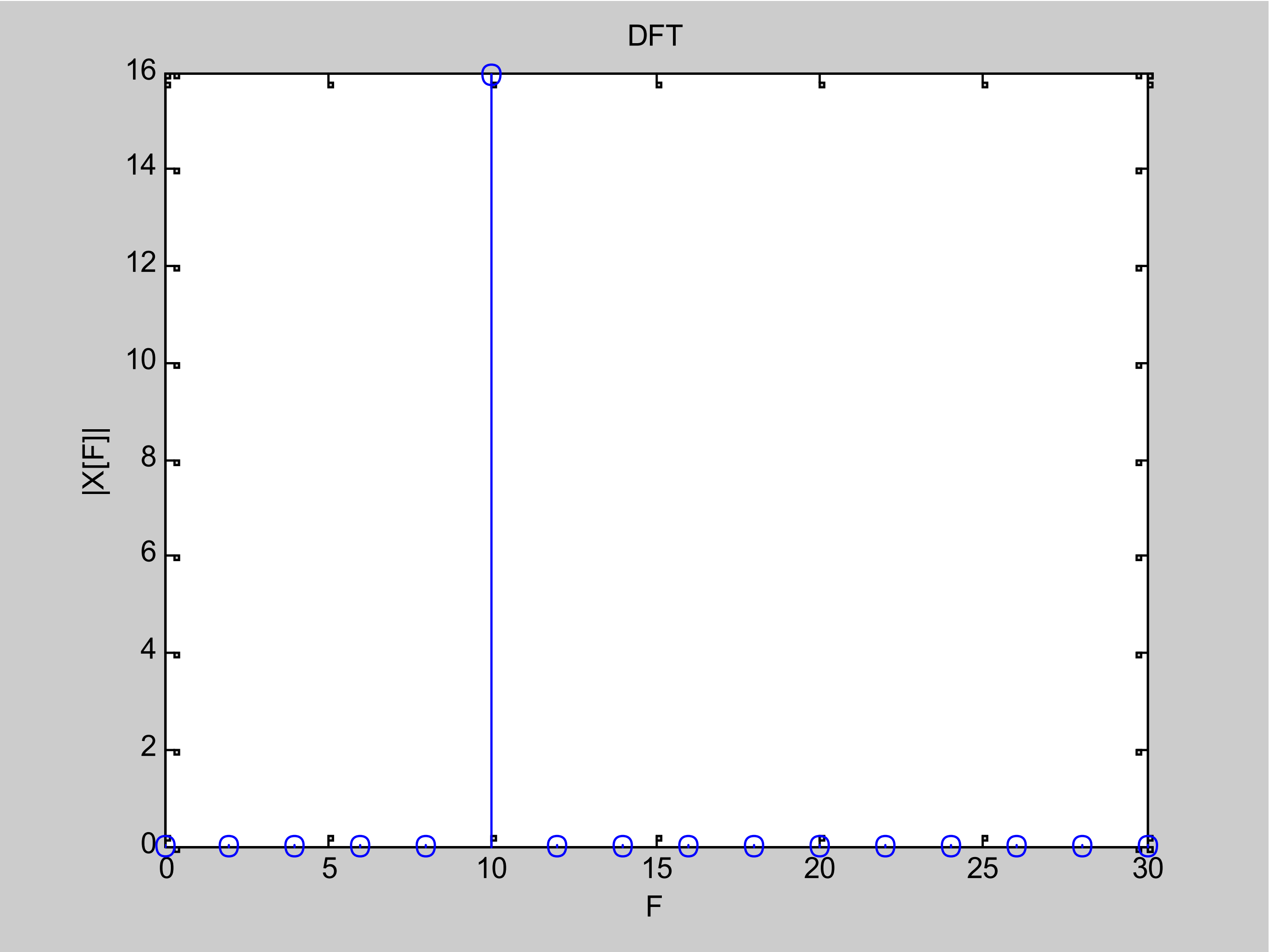
e.g.

F=10 Hz   Fs = 64 Hz   N = 32



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Relationship of DFT with Z - Transform

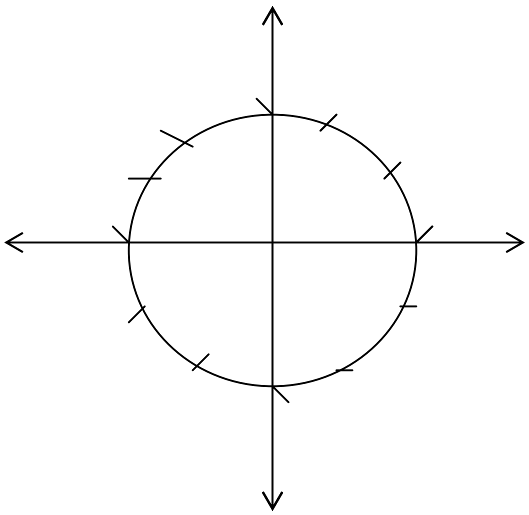
$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$z = re^{j\omega}$$

On unit circle

$$z = e^{j\omega}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$



Sample X(z) at ‘N’ equally spaced points on unit circle

$$X(z) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}$$

$$z_k = e^{j2\pi nk/N}$$

k = 0,1,……..,N

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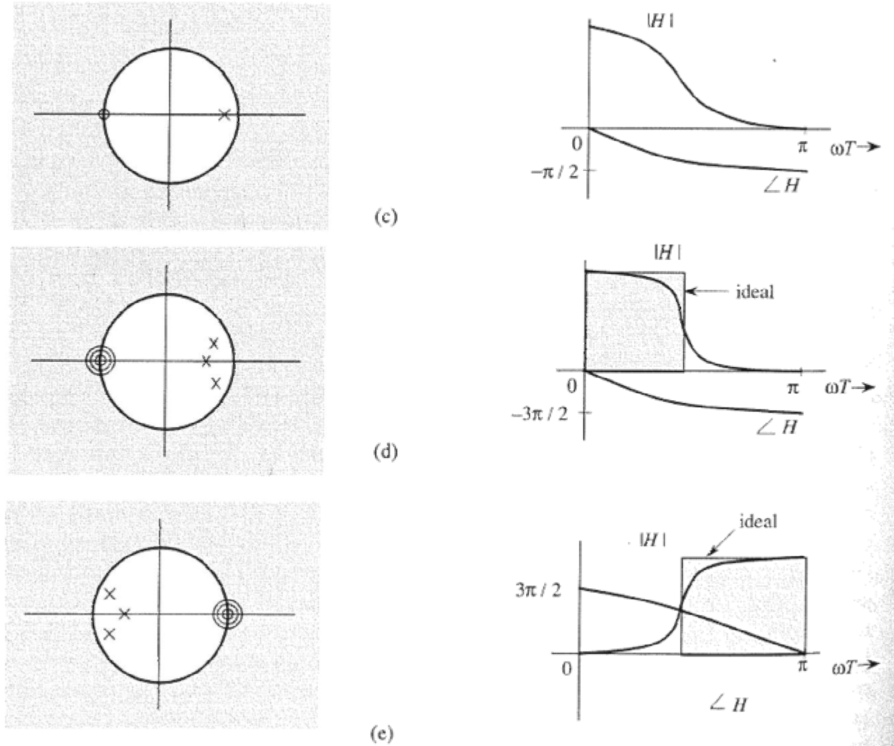
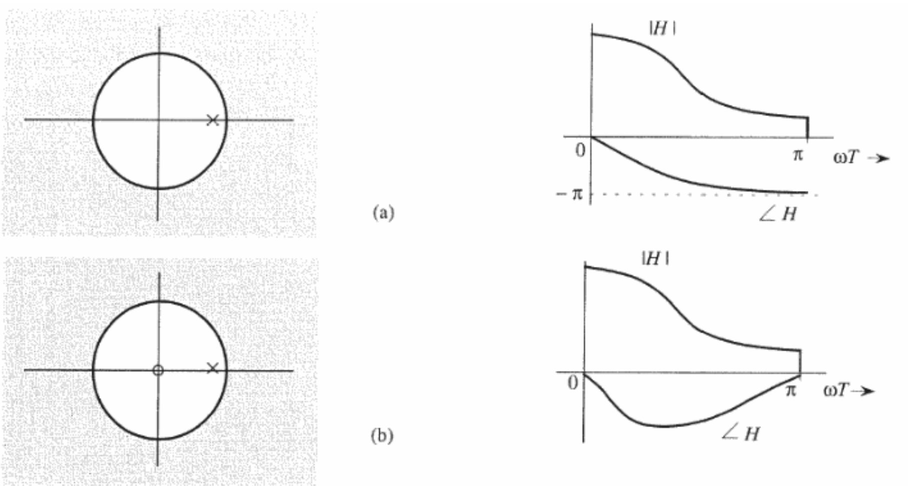
Controlling Gain by Pole-Zero Placement

- Frequency response can be controlled by assigning poles and zeros at specific locations.
- To enhance the gain at a frequency, we assign a pole at that frequency.
- To decrease the gain at a frequency, we assign a zero at that frequency





Spectral Analysis of Sinusoidal signals
or
Deterministic Signals
or
Stationary Signal

Signal length or Window length → N

<u>Effect of:</u>

DFT length → R

Window function

x(n) = 0.5sin(2*pi*n*f1) + sin(2*pi*n*f2)

f1 = 0.22 f2 = 0.34

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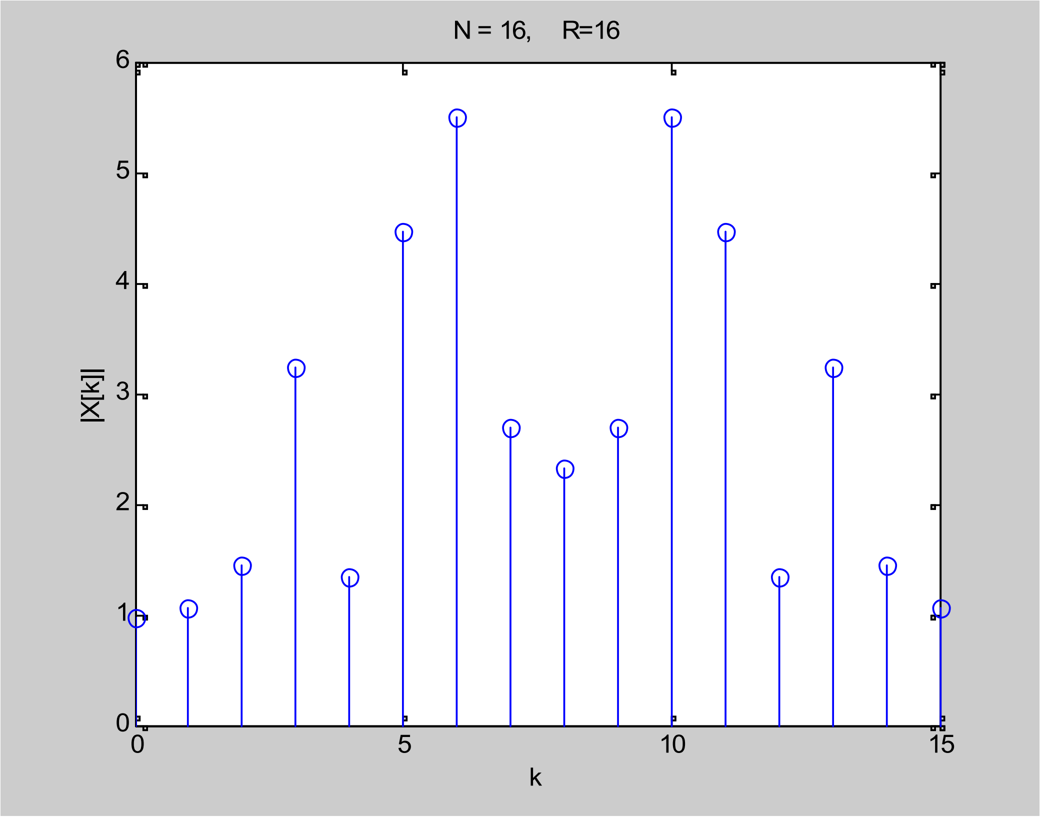
Effect of increasing length of DFT R by zero padding

Window function : Rectangular



Difficult to determine whether there is one or more sinusoids and their exact locations.

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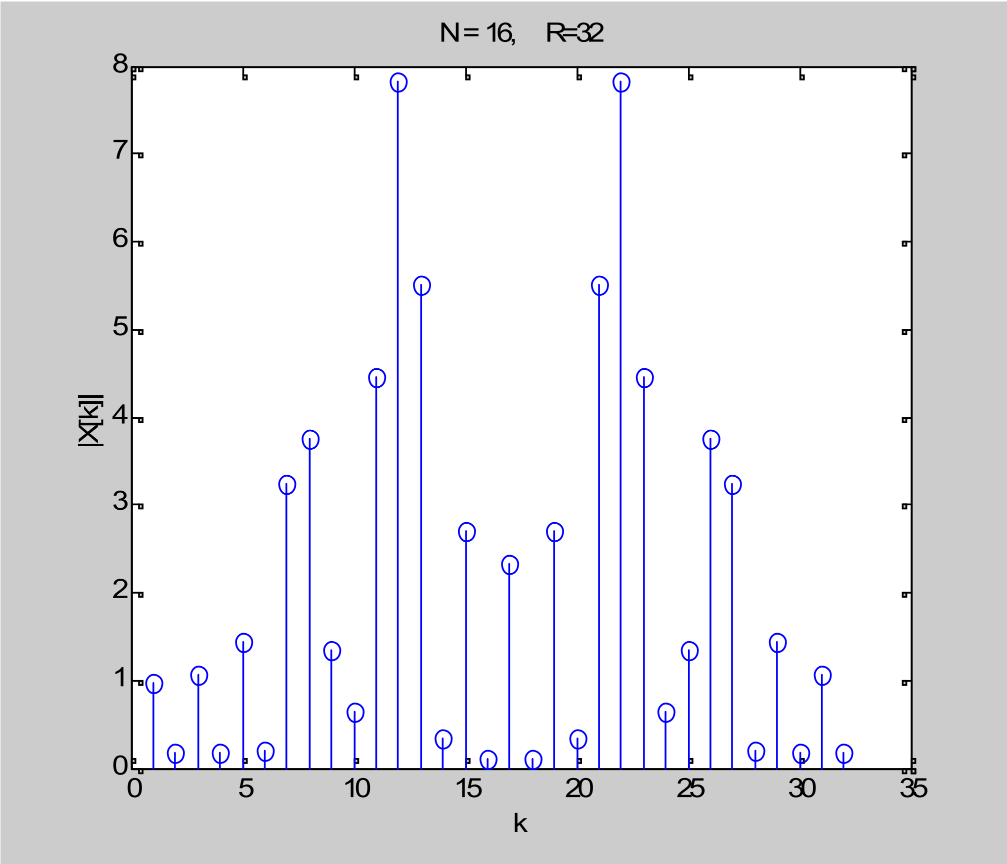

Size of DFT is increased
(by zero padding)
There is some concentration
Around k=7, & k=11

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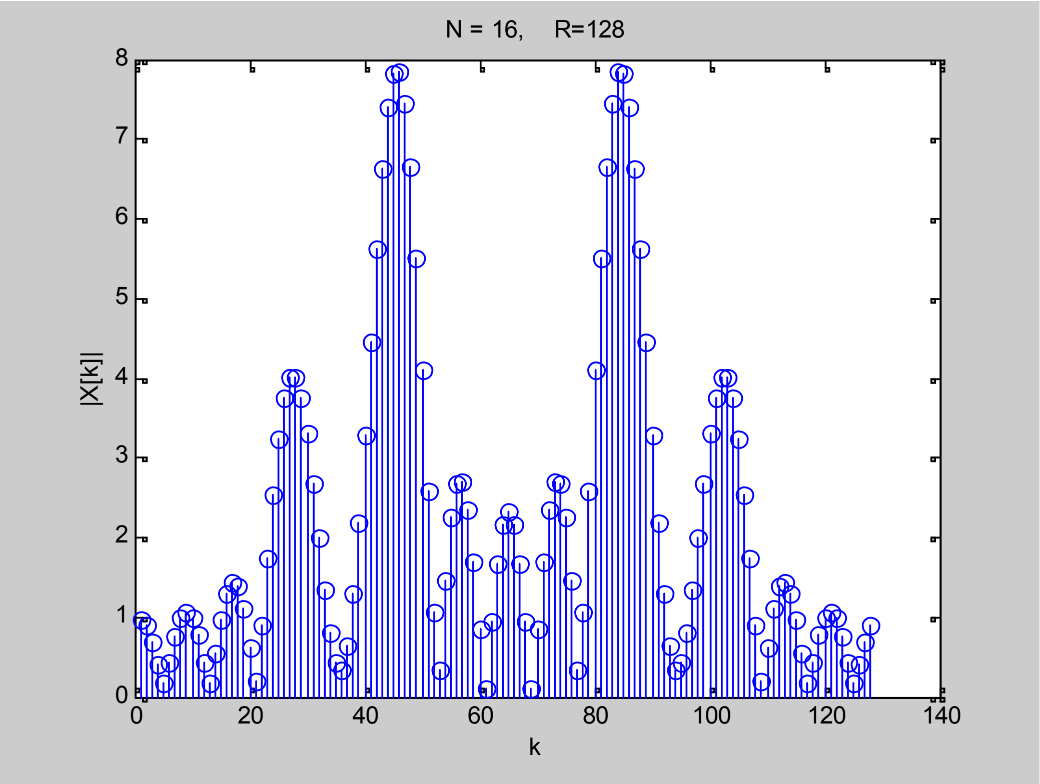

Peaks around k=27 & k=45
No. of minor peaks

Not clear whether additional sinusoids of lesser strengths are present

Increasing R improves sampling accuracy of DTFT reducing spectral separation of adjacent DTFT samples

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- Frequency Resolution Property
- Minimization of leakage using a tapered window

$$x(n) = 0.85\sin(2*pi*n*f1) + \sin(2*pi*n*f2)$$

$f1 = 0.22$ $f2 = 0.26$

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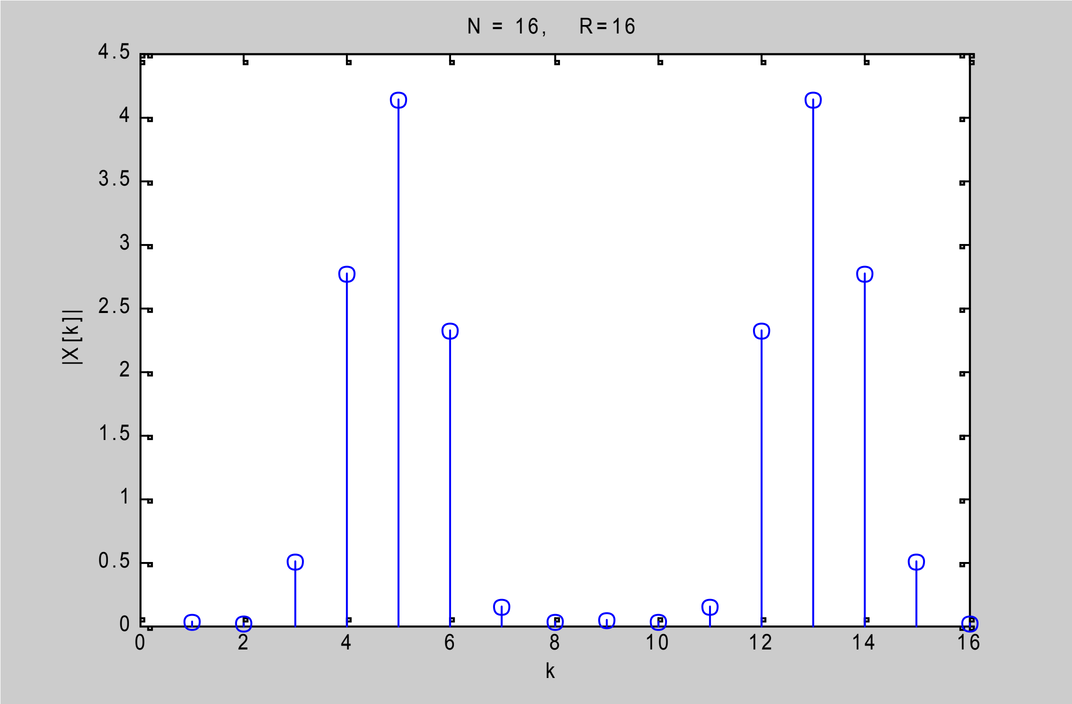

Leakage is minimized due to windowing

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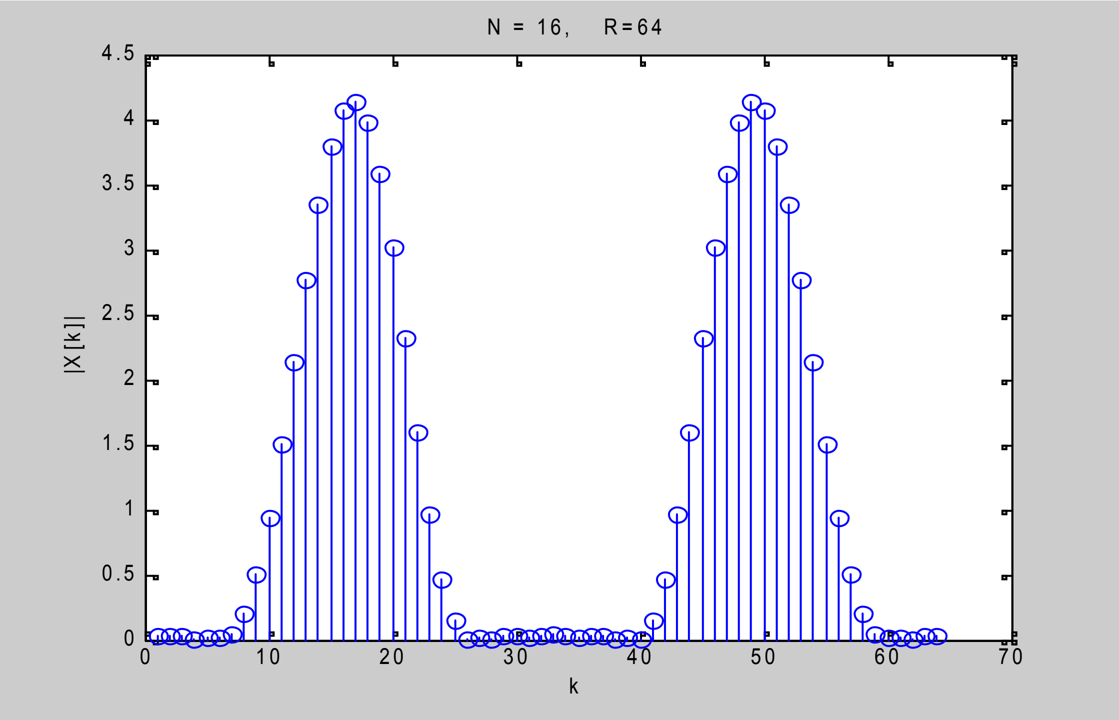

Increasing R,
No change in resolution

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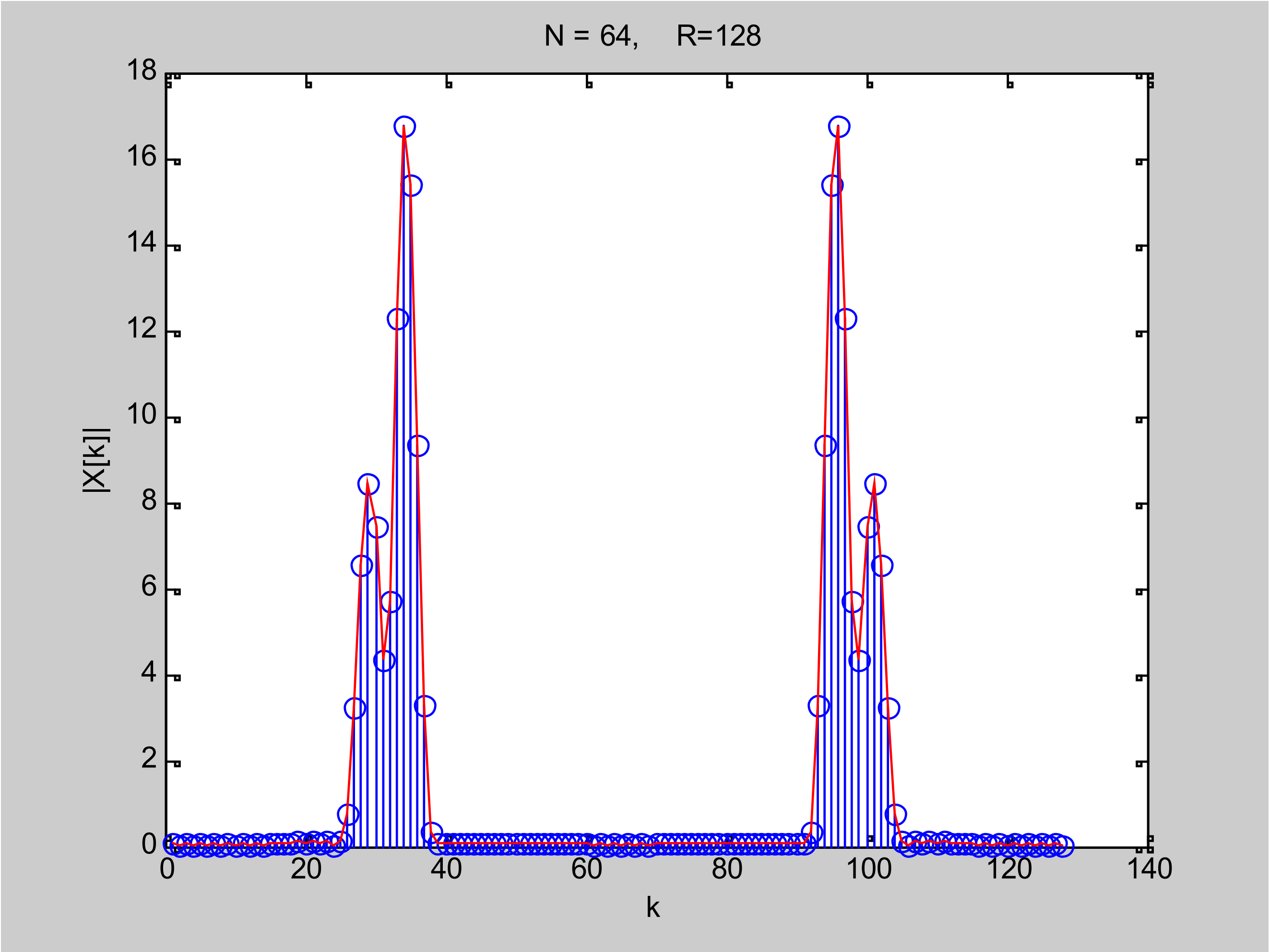

Increasing the window length improves the frequency resolution i.e. it can resolve two closer frequencies.

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Hamming window:
Width of main lobe : 8*pi/N

We can resolve two frequencies, if their difference is close to (greater than) half the main lobe width
i.e. 8*pi/2*N

Frequency difference is 0.04

For N=16, it is 0.125 (8*pi)/(2*pi*N*2)

For N=64, it is 0.03125

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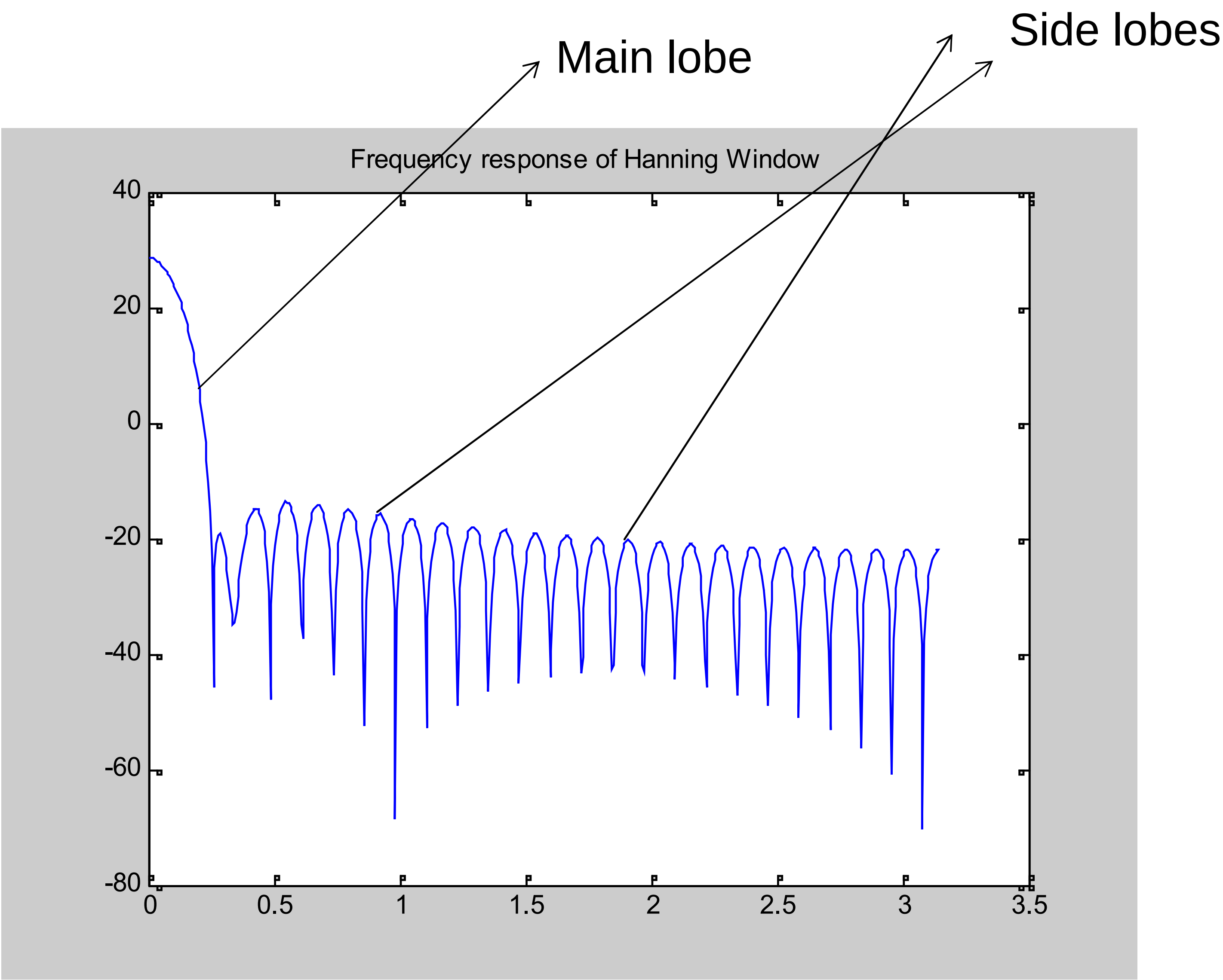

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Non-Stationary signals

Signals that change with time.

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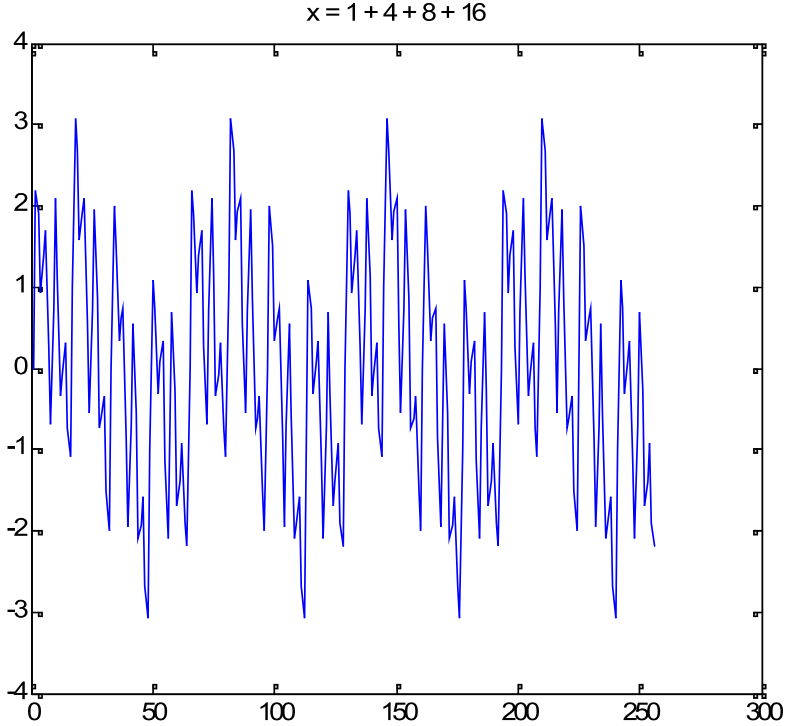
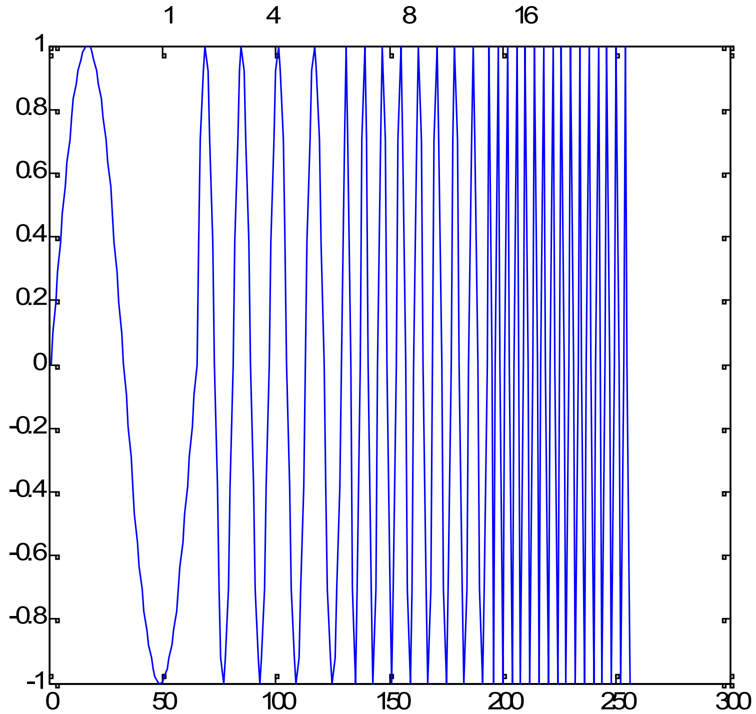
Stationary



Non-Stationary



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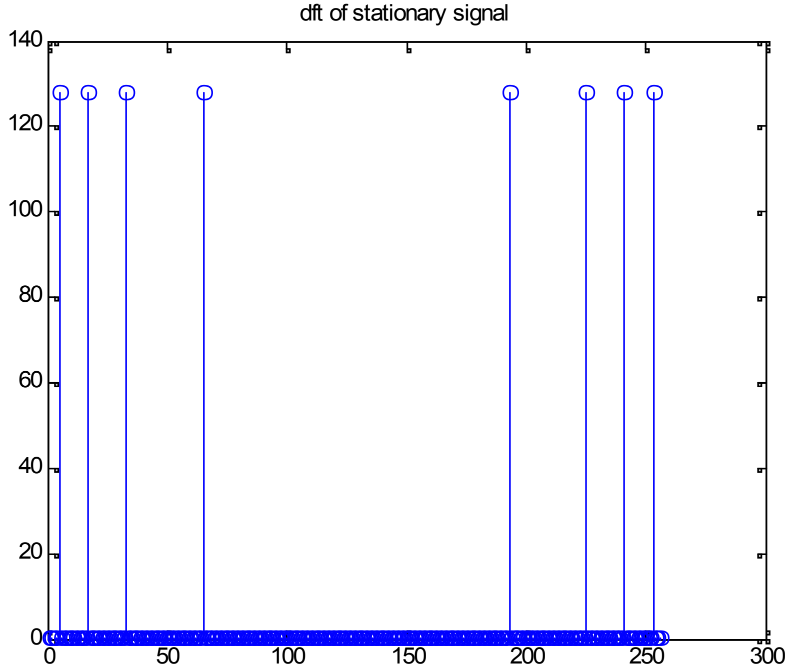

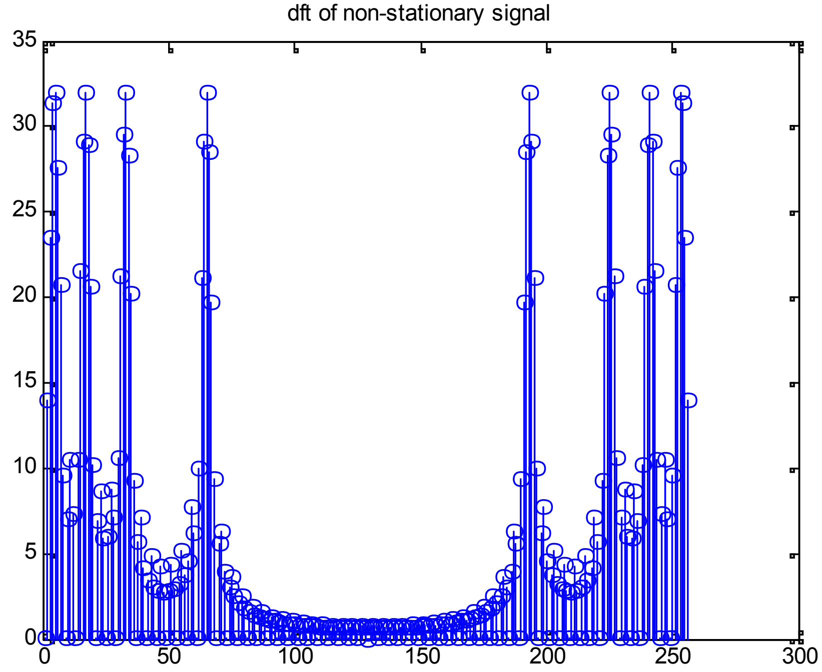

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Spectral Analysis of Random Signals

Estimate Power Density Spectrum

Periodogram Method

Signal → x(n) sectioned into smaller blocks
Window → w(n)
Modified blocks g(n) = x(n) x w(n)
Compute DFT of each modified block
Take square of magnitudes (Periodogram)
Take average of all periodograms

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