

1. Find  $\frac{dy}{dx}$  if  $(\cos y)^x = (\sin x)^y$

2. If  $f(x, y) = 0$  and  $\phi(x, z) = 0$  then show that

$$\frac{\partial \phi}{\partial x} \frac{\partial f}{\partial y} \frac{dy}{dz} = \frac{\partial f}{\partial x} \frac{\partial \phi}{\partial z}$$

3. If  $u = x^4 y^3$  where  $x = t^2$  &  $y = t^3$  then find  $\frac{du}{dt}$   $\{17t^{16}\}$

4. If  $z = f(x, y)$  &  $x = e^u + e^{-v}$ ,  $y = e^{-u} - e^v$  show that

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$$

2. If  $z = f(x, y)$  &  $x = u \cosh v$ ,  $y = u \sin hv$  then prove that

$$\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial u}\right)^2 - \frac{1}{u^2} \left(\frac{\partial z}{\partial v}\right)^2$$

3. If  $z = f(x, y)$  &  $u = e^x$ ,  $v = e^y$  show that  $\frac{\partial^2 z}{\partial x \partial y} = uv \frac{\partial^2 z}{\partial u \partial v}$

4. If  $u = f(x^2 - y^2, y^2 - z^2, z^2 - x^2)$  then prove that

$$\frac{1}{x} \frac{\partial u}{\partial x} + \frac{1}{y} \frac{\partial u}{\partial y} + \frac{1}{z} \frac{\partial u}{\partial z} = 0$$

5. If  $z = f(x, y)$ ,  $x = u \cos \alpha - t \sin \alpha$

$y = u \sin \alpha + t \cos \alpha$  Then prove that

$$(1) \left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial t}\right)^2 = \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$$

$$(2) \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial t^2}$$

6. If  $x = e^u \tan v$  &  $y = e^u \sin v$  then find the value of

$$\left(x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}\right) x \left(\frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y}\right)$$

7. If  $z = f(u, v)$  &  $u = 1x + my$ ,  $v = 1y - mx$  then show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (1^2 + m^2) \left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$$

8. If  $x = \sqrt{vw}$ ,  $y = \sqrt{wu}$ ,  $z = \sqrt{uv}$  prove that

$$u \frac{\partial \phi}{\partial u} + v \frac{\partial \phi}{\partial v} + w \frac{\partial \phi}{\partial w} = x \frac{\partial \phi}{\partial x} + y \frac{\partial \phi}{\partial y} + z \frac{\partial \phi}{\partial z}$$

Where  $\phi$  is a function of  $x, y, z,$ .

9. If  $u = x \log(xy)$  &  $x^3 + y^3 + 3xy = 1$  find  $\frac{du}{dx}$

10. If  $f(x, y) = x^4 y^2 \sin^{-1} \frac{y}{x}$  then show that

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = 6f$$

11. If  $u = \operatorname{cosec}^{-1} \left( \sqrt{\frac{\frac{1}{x^2} + \frac{1}{y^2}}{\frac{1}{x^3} + \frac{1}{y^3}}} \right)$  then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12} \left( \frac{13}{12} + \frac{\tan^2 u}{12} \right)$$

12. If  $u = \cos^{-1} \left( \frac{xy + yz + zx}{x^2 + y^2 + z^2} \right)$  then find the value of

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$$

13. If  $u = \log(x^3 + y^3 - x^2 y - yx^2)$  then find the value of

$$x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy}$$

14. If  $u = \log r$  where  $r = \sqrt{x^2 + y^2}$  then show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + 1 = 0$$

(15) If  $u = \sec^{-1} \left( \frac{x^3 - y^3}{x + y} \right)$  then find the value of  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$

16. If  $u = \sin^{-1} (x^2 + y^2)^{\frac{1}{5}}$  then prove that

$$(a) \quad x \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \frac{2}{5} \tan u$$

$$(b) \quad x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{2}{25} \tan u (2 \sec^2 u - 5)$$

17. If  $u = \frac{1}{3} \log \left( \frac{x^2 + y^2}{x - y} \right)$  then find the value of  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$

18. If  $u = x \sin^{-1} \frac{y}{x} + y \tan^{-1} \frac{x}{y}$  then find the value of  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$

19. If  $u = \cos^{-1} \left( \frac{x + y}{\sqrt{x} + \sqrt{y}} \right)$  then prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -\frac{1}{2} \cot u$

20. If  $u = \log \left( \frac{x^2 y^2}{x + y} \right)$  find  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$

21. If  $u = \sin^{-1} \left( \frac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{5}} + y^{\frac{1}{5}}} \right)$  prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{20} \tan u$

And  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{400} (\tan^2 u - 19)$

22. If  $u = \frac{x^2 + y^2 + z^2}{x + y + z}$  then verify Euler's theorem on homogenous functions.

23. If  $z=f(x, y)$  and  $u, v$ , are function of degree  $n$  in  $x$  &  $y$  then show that

$$u \frac{\partial z}{\partial u} + v \frac{\partial z}{\partial v} = \frac{1}{n} \left( x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} \right)$$